

$$13a) \underbrace{(1-y \sin x)}_{=P=\frac{\partial F}{\partial x}} + \underbrace{\cos x \cdot y'}_{=Q=\frac{\partial F}{\partial y}} = 0$$

$$\frac{\partial F}{\partial y} = -\sin x = \frac{\partial Q}{\partial x} \rightarrow \text{exact.} \quad (4P)$$

$$F = \int (1 - y \sin x) dx = x - y(\cos x) + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = \cos x = \cos(x) + h'(y) \Rightarrow h'(y) = 0 \Rightarrow h(y) = c \quad (2P)$$

$$\text{Solution: } C_1 = x - y \cdot \cos(x) + c \Rightarrow y = \frac{D-x}{\cos(x)} \quad (2P)$$

$$\underbrace{P=\frac{\partial F}{\partial x}}_{=Q=\frac{\partial F}{\partial y}} = \underbrace{Q=\frac{\partial F}{\partial y}}_{=Q=\frac{\partial F}{\partial y}} \quad (D=C-c)$$

$$b) \frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}} \cdot y' = 0$$

$$\frac{\partial F}{\partial y} = \frac{x-y}{\sqrt{x^2+y^2}} = \frac{\partial Q}{\partial x} \rightarrow \text{exact.} \quad (4P)$$

$$F = \int \frac{x}{\sqrt{x^2+y^2}} dx = \sqrt{x^2+y^2} + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}} + h'(y) \Rightarrow h'(y) = 0 \Rightarrow h(y) = c \quad (2P)$$

$$\text{Solution: } C_1 = \sqrt{x^2+y^2} + c \Rightarrow y^2 = D - x^2 \quad (2P)$$

$$c) \underbrace{-x + (x-y)}_{=P=\frac{\partial F}{\partial x}} \cdot y' = 0$$

$$\frac{\partial F}{\partial y} = 0 \quad \frac{\partial Q}{\partial x} = 1 \rightarrow \text{not exact.} \quad (10P)$$

$$d) \underbrace{-(2-\frac{y}{x})}_{=P=\frac{\partial F}{\partial x}} + \underbrace{\ln|x| y'}_{=Q=\frac{\partial F}{\partial y}} = 0$$

$$\frac{\partial F}{\partial y} = \frac{1}{x} = \frac{\partial Q}{\partial x} \rightarrow \text{exact} \quad (4P)$$

$$F = \int -2 + \frac{y}{x} dx = -2x + y \ln|x| + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = \ln|x| = \ln|x| + h'(y) \Rightarrow h'(y) = 0 \Rightarrow h(y) = c \quad (2P)$$

$$\text{Solution: } C_1 = -2x + y \ln|x| + c \Rightarrow y = \frac{D+2x}{\ln|x|} \quad (2P)$$

$$e) \underbrace{2x^2y + 4x^3}_{=P=\frac{\partial F}{\partial x}} + \underbrace{(2x^2y + 2y)}_{=Q=\frac{\partial F}{\partial y}} \cdot y' = 0$$

$$\frac{\partial F}{\partial y} = 4xy = \frac{\partial Q}{\partial x} \rightarrow \text{exact} \quad (4P)$$

$$F = \int (2x^2y + 4x^3) dx = x^2y^2 + x^4 + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = 2x^2y + 2y = 2x^2y + h'(y) \Rightarrow h'(y) = 2y \Rightarrow h(y) = y^2 + c \quad (2P)$$

$$\text{Solution: } C_1 = x^2y^2 + x^4 + y^2 + c \Rightarrow y^2 = \frac{D-x^4}{1+x^2} \quad (2P)$$

$$14a) \underbrace{\frac{y^2-x^2}{x^2y^2}}_{=P=\frac{\partial F}{\partial x}} + \underbrace{\frac{1}{y^2} y'}_{=Q=\frac{\partial F}{\partial y}} = 0 \quad (2P)$$

$$\left(\frac{\partial F}{\partial y} = \frac{1}{y^2} = \frac{\partial Q}{\partial x} \right)$$

$$F = \int \frac{1}{x} - \frac{1}{y} dx = \ln|x| - \frac{x}{y} + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = \frac{x}{y^2} + h'(y) = \frac{x}{y^2} \Rightarrow h'(y) = 0 \Rightarrow h(y) = C. \quad (2P)$$

solution:

$$C = \ln|x| - \frac{x}{y} + C = b \Rightarrow y = \frac{x}{\ln|x| + b} \quad (4P) \quad (D=C-C)$$

$$b) \quad \frac{\partial F}{\partial x} = \frac{1}{x} = \frac{\partial F}{\partial y} = \frac{x}{y^2} \Rightarrow \frac{1}{x} = \frac{x}{y^2} \Rightarrow y^2 = x^2 \Rightarrow y' = 0 \quad (2P)$$

$$\left(\frac{\partial F}{\partial y} = \frac{1}{x^2} = \frac{\partial F}{\partial x} \right)$$

$$F = \int \frac{1}{x^2} dx = -\frac{1}{x} + h(y) \quad (2P)$$

$$\frac{\partial F}{\partial y} = -\frac{1}{x} + h'(y) = -\frac{1}{x} \Rightarrow h'(y) = 0 \Rightarrow h(y) = \frac{y^2}{2} + C \quad (2P)$$

Solution:

$$C_1 = -\frac{1}{x} + \frac{1}{2}y^2 + C \Rightarrow D = \frac{1}{2}y^2 - \frac{1}{x} = 0 \Rightarrow y = \frac{1}{x} \pm \sqrt{2D + \frac{1}{x^2}} \quad (4P)$$

15.) A) and IV) (6P)

$$(1 + 2xy' = 0 \rightarrow \text{exact})$$

B) and II) (6P)

$$(2x + y + xy' = 0 \rightarrow \text{exact})$$

C) and I) (6P)

$$(y^2 + 2xy' = 0 \rightarrow \text{exact})$$

D) and V) (6P)

$$(1 + y + (1+y)x y' = 0 \rightarrow \text{exact})$$

E) and III) (6P)

$$\left(\frac{1}{x} + \frac{1}{y} y' = 0 \rightarrow \text{exact} \right)$$

$$F) \quad \sqrt{1+x^2+y^2} \geq 1$$

$$\Rightarrow y' \geq 1$$

• no maximal/minimal ($y' \neq 0$) (4P)

• a single zero at $x=0$ ($y(0)=0$), since (4P)

$y(x)$ monotonic increasing

• $y(x) \rightarrow +\infty$ if $x \rightarrow +\infty$ since $y' \geq 1$. (4P)